Final Exam

EE313 Signals and Systems

Fall 1999, Prof. Brian L. Evans, Unique No. 14510

December 11, 1999

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework and solution sets.
- Calculators are allowed.
- You may use any standalone computer system but it may not be connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- Fully justify your answers unless instructed otherwise.

Problem	Point Value	Your Score	Topic
1	15		Difference Equation
2	10		Discrete-Time Convolution
3	10		Tapped Delay Line
4	15		Continuous-Time Stability
5	15		Sigma-Delta Modulation
6	10		Fourier Series
7	15		Modulation
8	10		Potpourri
Total	100		

M-3

1. Difference Equation

Solve the following difference equation

$$y[n] + 3/4\, y[n-1] + 1/8\, y[n-2] = u[n]$$

with the initial conditions $y[-2] = 0$ and $y[-1] = 0$ by using the z-transform. Note that $u[n]$ is the unit step function.

M-4

2. Discrete-Time Convolution

Sketch the result of the following convolutions. On the sketches, clearly label significant points on the n and $y[n]$ axes. You do not have to show intermediate work, e.g. the flip-and-slide method, but showing intermediate work may qualify for partial credit.

a. $y[n] = p[n] * p[n]$, where

$$p[n] = \begin{cases} 1 & \text{for } 0 <= n < N \\ 0 & \text{otherwise} \end{cases}$$

b. $y[n] = u[-n] * u[-n]$, where $u[n]$ is the unit step function.

3. Tapped Delay Line.

A tapped delay line is a linear time-invariant system. In continuous time, the output signal $y(t)$ to an input signal $x(t)$ is given by

$$y(t) = \sum_{n=0}^{N-1} a_n \, x(t - nT)$$

a. Sample the tapped delay using a sampling period of T seconds and write the corresponding equation for $y[k]$:

b. For the sampled tapped delay line, what is the impulse response?

c. What is another name for the sampled tapped delay line?

d. What is the relationship between the transfer function of the continuous-time tapped delay line and the sampled tapped delay? That is, what is the mapping from the z-domain to the s-domain?

4. Continuous-Time Stability.

Given a linear time-invariant continuous-time system with input $f(t)$ and output $y(t)$ described by the following differential equation

$$y''(t) + 3\,y'(t) + K\,y(t) = f(t)$$

where K is a real-valued parameter.

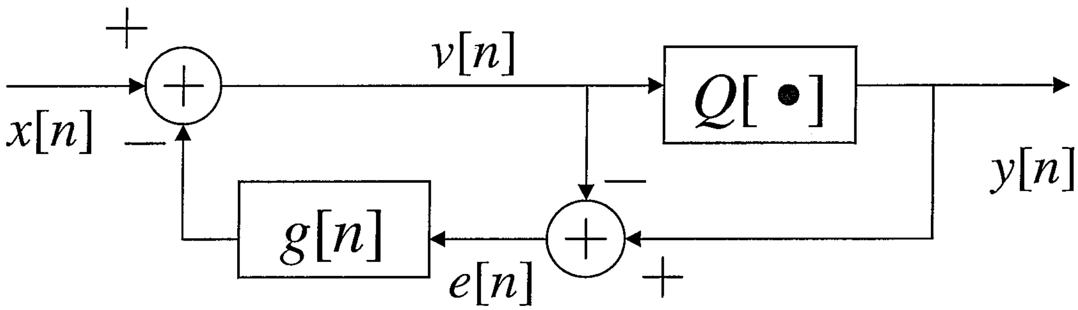
a. What are the characteristic roots?

b. For what range of K makes the system stable?

M-7

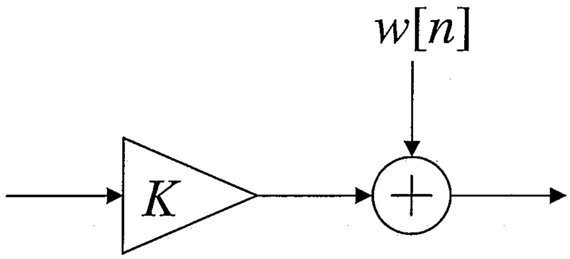
5. Sigma-Delta Modulation.

Shown below is a type of sigma-delta modulator called a noise-shaping feedback coder.



Here, $g[n]$ is the impulse response of an LTI system. The quantizer Q outputs 1 if the input is greater than or equal to 0, and –1 otherwise.

One way to linearize the system is to approximate the quantizer as a gain K in cascade with an additive noise signal $w[n]$, as shown on the right. Based on this model of the quantizer:

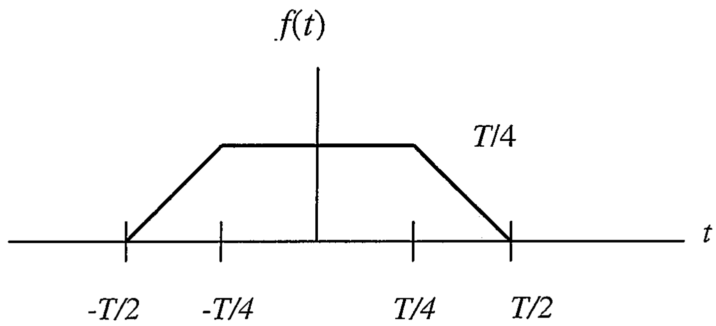


a. Derive the signal transfer function from $x[n]$ to $y[n]$ by setting $w[n] = 0$.

b. Derive the noise transfer function from $w[n]$ to $y[n]$ by setting $x[n] = 0$.

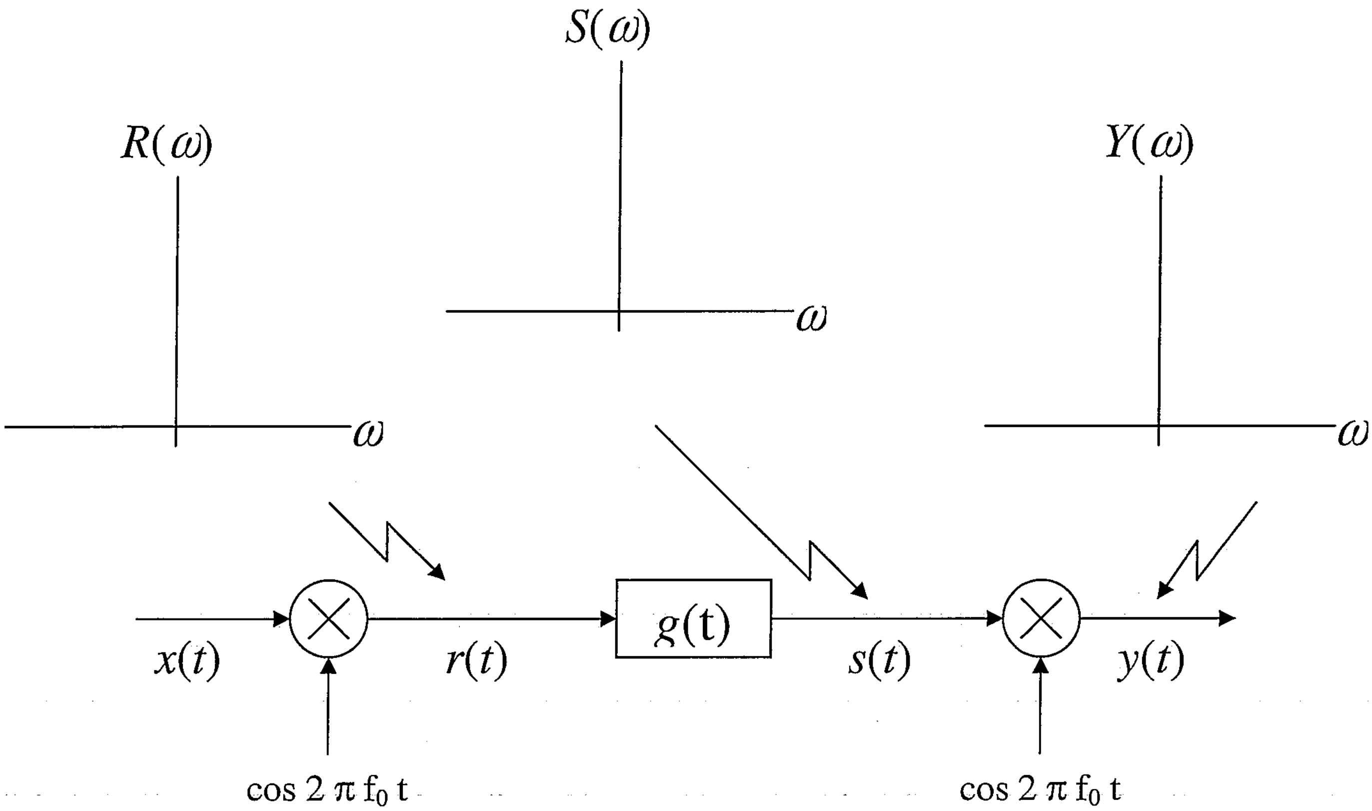
6. Fourier Series

Compute the Fourier Series of the following waveform $f(t)$, which has a period of T:

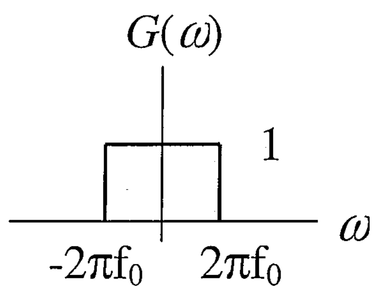


7. Modulation

Sketch the Fourier transform of $r(t)$, $s(t)$, and $y(t)$ in the following cascade given that the input signal $x(t) = \delta(t)$ and that $g(t)$ is the impulse response of a lowpass filter



such that



What type of a filter does the overall system implement? Allpass, lowpass, bandpass, highpass, or bandstop?

8. Potpourri

Answer True or False to the following questions. If False, then write a brief justification or provide a counterexample.

a) Consider a system that modulates the input signal by $\cos(2\pi f_0 t)$ to produce the output signal. This system obeys the Fundamental Theorem of Linear Systems.

b) Considering all possible systems, Laplace and Fourier transforms may only be applied to linear time-invariant systems.

c) Analog-to-digital converters essentially consist of a cascade of a lowpass anti-aliasing filter, a sampling device, and a quantizer. Digital-to-analog converters essentially consist of a discrete-to-continuous device and a lowpass anti-imaging filter.

d) Convolution of two continuous-time signals $x(t)$ and $y(t)$ may always be computed by taking the Laplace transforms of $x(t)$ and $y(t)$, multiplying the Laplace transforms together, and inverse Laplace transforming the result.

e) In a discrete-time filter, the location of zeros determines the passband(s) in the magnitude response, and the location of the poles determines the stopband(s) in the magnitude response.